SPECIFICATION

TITLE OF THE INVENTION

Methods of Designing Optimal Linear Controllers

TECHNICAL FIELD OF THE INVENTION

This invention relates to the design of optimal multivariable linear controllers and PID controllers.

BACKGROUND OF THE INVNTION

A PID (proportional-integral-derivative) controller is a device, usually implemented in a computer, that is used to control a process variable (e.g., temperature, pressure, etc) of an industrial process. For a multiple-input and multiple-output (MIMO) process, the process variable y(t) and controller output u(t) are n-dimensional vector variables of the time t. The controller receives the sample values of y(t) and calculates its output u(t) according to a PID equation. The controller sends its output u(t) to the process so that the error e(t)=r(t)-y(t) approaches zero as time t increases, where r(t) is the set point (also known as the reference signal or the command signal, etc).

There are many types of PID controllers, depending on the use of different types of PID equations. In discrete time form, all these PID controllers can be viewed as special cases of the general linear controller with the linear control equation

$$Du_k=Er_k-Cy_k$$

where $y_k=y(t_k)$ is the process variable at time $t_k=t_0+kT_s$, t_0 is the initial time, $T_s>0$ is the constant sampling period, k=0, 1, 2, ... is a non-negative integer called discrete time variable, $u_k=u(t_k)$ is the controller output at time t_k, and u_k can be subject to upper limit

and/or lower limit constraints placed on one or more of its components, $r_k=r(t_k)$ is the set point at time t_k, D, E and C are n by n matrix polynomials in the unit backward shifting operator z^{-1} such that for any discrete time signal x_k, $z^{-1}x_k=x_{k-1}$, and one or more of the said D, E and C contain tuning parameters that are to be determined,

For example, if $D=(1-z^{-1})/T_s*I$, where I is the identity matrix of order n, and $E=C=K_i+K_pD+K_dD^2$, where K_i, K_p and K_d are user specified constant coefficient matrices called the integral gain, the proportional gain and the derivative gain, then the linear controller becomes the following "type A" PID controller

$$Du_k=K_ie_k+K_pDe_k+K_dD^2e_k$$

where $e_k=r_k-y_k$ is the error. The three gains K_i, K_p and K_d are the tuning parameters.

If the aforesaid E is changed to K_i+K_pD or K_i, then the linear controller becomes the following "type B" and "type C" PID controllers, respectively:

$$Du_k=K_ie_k+K_pDe_k-K_dD^2y_k$$

$$Du_k=K_ie_k-K_pDy_k-K_dD^2y_k$$

Once its structure is selected, the performance of a linear controller or a PID controller depends mainly on the choice of its tuning parameters.

How to properly choose the tuning parameters for a PID controller is a problem that has attracted a lot of studies ever since PID controllers became widely used in industry in the early 1940s. The Ziegler-Nichols tuning methods developed by Ziegler and Nichols in 1942 and 1943 are still widely used in industry. Other model based methods choose the tuning parameters by minimizing some well-known control performance index such as the

integral absolute errors (IAE), the integral squared errors (ISE), etc. However, practice shows that all these methods often lead to oscillatory control results.

DETAILED DESCRIPTION OF THE INVENTION

Suppose the open-loop transfer function of the process from u_k to y_k is $A^{-1}B$, where A and B are known n by n matrix polynomials in the unit backward shifting operator z^{-1}:

$$A=I - A_1z^{-1} - A_2z^{-2} - \dots - A_pz^{-p}$$

$$B= B_1z^{-1} + B_2z^{-2} + \dots + B_qz^{-q}$$

where I is the identity matrix of order n, p and q are two known positive numbers, and A_1, A_2, ..., A_p, B_1, B_2, ..., and B_q are constant coefficient matrices of order n. A and B can be obtained by use of the commercially available software product "System Identification Toolbox" from The MathWorks Inc. Below is another way for finding A and B.

Select a positive integer v that is not less than n(p+q) and let k be not less than v + max{p, q}, where max{p, q} denotes the larger one of the p and q. Let h_k be a vector defined by

$$h_k^T=[y_{k-1}^T \quad y_{k-2}^T \dots \quad y_{k-p}^T \quad u_{k-1}^T \quad u_{k-2}^T \dots \quad u_{k-q}^T \quad 1]$$

where for any matrix X, X^T denotes the transpose of X, and let

$$Y=[y_k \quad y_{k-1} \dots \quad y_{k-v}]^T$$

$$H=[h_k \quad h_{k-1} \dots \quad h_{k-v}]^T$$

$$W=\mathrm{diag}(1, w, w^2, \dots, w^v)$$

where w is a positive real number not larger than one, and W is a diagonal matrix with 1, w, w^2, ..., and w^v on its diagonal. The number k should be large enough and the controller

output signal u_k should be chosen so as to ensure that the matrix H is of full column rank. Then the matrices $A_1, A_2, \ldots, A_p, B_1, B_2, \ldots$, and B_q, and the vector d can be obtained from

$$[A_1 \quad A_2 \ldots A_p \quad B_1 \quad B_2 \ldots B_q \quad d] = ((WH)^+(WY))^T$$

where for any matrix X, X^+ denotes the Moore-Penrose pseudo-inverse of X, which is a standard function under the name "pinv" in the software product MATLAB from The MathWorks Inc. (MATLAB is a registered trademark of The MathWorks Inc.). Thus the two polynomials A and B and a vector d are obtained. When A, B, and d are so obtained, the process can be approximately represented by the following linear model:

$$Ay_k=Bu_k+d$$

And the open-loop transfer function of the process is then $A^{-1}B$. Please note that this invention does not deal with the problem of how to get the polynomials A and B. From now on it is always assumed that, one way or another, the two polynomials A and B are known.

If the process is controlled by the following linear controller as described in the previous section:

$$Du_k=Er_k-Cy_k$$

where D, E, and C are n by n polynomials in the backward shifting operator z^{-1}, and one or more of D, E and C contain tuning parameters that are to be specified, then the closed-loop transfer function from the set point r_k to the process variable y_k is

$$Q=(A+BD^{-1}C)^{-1}BD^{-1}E$$

From Q the characteristic polynomial in z, denoted by b(z), can be found. All roots of the characteristic equation b(z)=0, denoted by z_1, z_2, z_3, ..., z_j, where j is a positive number, form all the poles of Q.

Let max $\{|z_1|, |z_2|, \ldots, |z_j|\}$ denote the maximum of all the absolute values $|z_1|$, $|z_2|$, ..., and $|z_j|$ of the q poles of Q. Then the best choice of the tuning parameters is such that the max $\{|z_1|, |z_2|, \ldots, |z_j|\}$ is minimized, i.e., the best tuning parameters form a solution to the following minimax problem:

$$\min \max \{|z_1|, |z_2|, \ldots, |z_j|\}$$

In situations where one or more of the tuning parameters must be within user specified regions, the above minimax problem becomes a constrained minimax problem.

Many commercially available software products can directly be used to find the poles of a transfer function and the roots of a polynomial, and to solve the minimax or constrained minimax problem as formulated above, such as (1) the function "pole" in the "Control System Toolbox", which can find the poles directly from the transfer function Q, (2) the function "roots" in MATALAB, which can directly find the roots of the characteristic equation b(z)=0, and (3) the functions "minimax" and "fminimax" in the "Optimization Toolbox", which directly provides solution to the constrained or unconstrained minimax problem formulated above, all of which are easy to use and commercially available from The MathWorks Inc.